

Debye Screening mass from effective Potentials in Lattice gauge theory

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February 1, 2008

Abstract

In this paper, we use a definition of the Debye screening mass in $SU(2)$ pure gauge theory at high temperature in $3+1$ dimensions, that makes use of the walls between different Z_2 domains. We show, that this definition enables us to unambiguously obtain the Debye Screening mass from the curvature of the effective potential for the Polyakov loops calculated in the modified axial gauge. Using an approach on asymmetric lattices, we obtain numerical values for the Debye mass and compare them to perturbative as well as Monte-Carlo results. Our resulting Debye-Masses are significantly higher than the one-loop perturbative one. They are within the errors in agreement with those obtained by Monte-Carlo methods. Open questions are discussed.

1 Introduction

The Debye Screening mass in high Temperature $SU(2)$ pure gauge theory has been subject of continued interest. The perturbative value in one loop perturbation theory is known from calculations of the vacuum polarization tensor [1] as well as other observables.

The perturbative definition of screening phenomena however remains unclear with regard to infrared singularities in higher order perturbation theory [2] as well as the convergence of the perturbation series in non-abelian gauge theories in general.

Numerical investigations [3] [4] show, that there is a significant discrepancy in the size of the Debye-Mass between calculations on a lattice using Monte-Carlo methods and the perturbative result. Up to now, it remains unclear, to what extent higher order calculations may overcome these differences or which non-perturbative effects might play a role in the process of screening in non-abelian gauge theories even at leading order.

It has been emphasized, that the Haar-Measure associated with the gauge group

$SU(2)$ plays a major role when calculating properties of the Polyakov loop in the modified axial gauge $\partial_0 A_0 = 0$ [5]. This importance of the Haar measure stems from the fact, that in the perturbative limit $g \rightarrow 0$ the non-abelian nature of the Gauge group is inevitably lost, $SU(2) \rightarrow U(1)^{N-1}$, thus effectively neglecting effects of the non-trivial Haar-measure. Since it is this Haar-Measure, which leads to non-trivial topological effects and thus to a feature, which is believed to be specific for a non-abelian Gauge theory, it is desirable to take into account its effects also in the high-temperature phenomenon of Debye-Screening.

This paper is organized as follows. After the Introduction in part one, we review a definition of the Debye-Mass via the profile of the domain wall between different Z_2 domains of the Polyakov-Loop, which fulfills the requirements of [6] and suits our approach well. Furthermore, we show its consistency with perturbative calculations in one loop perturbation theory. In the third part, we summarize some results of [7] and [8] on the method of asymmetric lattices, which will be used further on. The fourth part contains our numerical calculations of the curvature of the effective Potential for the Polyakov-Loop and the results in terms of the physical temperature. The fifth and last part is devoted to a summary of our results and a discussion of open problems.

2 Debye-Mass from the effective Potential of the Polyakov-Loops

There are various suggestions for a definition of the Debye Screening Mass, ranging from the exponential falloff of the correlator between two Polyakov loops to the static limit of the longitudinal vacuum polarization tensor. A requirement that has to be fulfilled by any of them to make sense is its property of being odd under time reflection [6]. The definition, that we want to use here has been suggested by [9] and fulfils the abovementioned requirements.

It uses the fact, that the center symmetry is broken in the high temperature phase in a $SU(2)$ gauge theory and the Polyakov loop may take on one of the two possible Z_2 -values. If there exist two different Z_2 phases in space, then a domain wall $p(z)$ will develop between them. They can be realized on a lattice by introducing a twist in the action and can thus serve as a laboratory for measuring the Debye-Mass.

The Debye-Mass is then given by the exponential falloff of the profile of the wall between different Z_2 domains, where here we will investigate only domain walls developping in the z direction $p(z)$

$$m_D = \lim_{z \rightarrow \pm\infty} \left| \frac{\partial_z p}{2p} \right|. \quad (1)$$

The physics of this gauge-invariant non-perturbative definition is clear in the abelian case. In the following, we will show in one loop perturbation theory, that it coincides with the usual perturbative result in $SU(2)$.

This goal is achieved by calculating the explicit solution in a Weiss-potential [10] extending in the z -direction and connecting two different Z_2 -domains.

Since in the modified axial gauge the vector potential $a_0 \sim \tau^3$ we have to deal with a simple $(a_0^3)^4$ -Theory, the field equation of which for a soliton is given by

$$a_0'' - \frac{\partial U}{\partial a_0}(z) = 0 \quad (2)$$

for the potential $U[a_0(z)] = \frac{a_0^4}{4} - \frac{m^2 a_0^2}{2} + \text{const.}$ The solution of this equation is

$$a_0(z) = \pm m \tanh[m(z - z_0)]. \quad (3)$$

The potential energy of this solution is

$$V[a_0] = \int dx \left(\frac{1}{2} (\nabla a_0)^2 + U(a_0) \right). \quad (4)$$

Using a functional Taylor expansion for small fluctuations $\eta(z)$ around the minima $(a_0)_0$ of the potential

$$V[a_0] = V[(a_0)_0] + \int dz \frac{1}{2} \left(\eta(z) \left[-\nabla^2 + \left(\frac{d^2 U}{da_0^2} \right)_{(a_0(z))_0} \right] \eta(z) + \dots \right), \quad (5)$$

one can see immediately, that the exponential falloff m of the solitonic solution (3) is given by the square root of the curvature of the potential at the minimum

$$m_D = \sqrt{\left. \frac{d^2 V_{eff}[a_0]}{da_0^2} \right|_{\text{Min}}}. \quad (6)$$

Calculating this property for the effective potential in one-loop perturbation theory [10], one gets the result

$$m_D = \sqrt{\frac{2}{3}} g T \quad (7)$$

which coincides with the one-loop result for the Debye-Mass as calculated with other definitions.

It is a peculiarity of the perturbative calculation, that the effect of the nontrivial Haar-measure is cancelled exactly by the contribution of the longitudinal gluons, thus leaving the range of integration in the Path integral for the a_0 unclear. Studies of effective actions allowing for field configurations with a nontrivial space-dependence [11][12] (as compared to the calculation of [10], using $a_0(x) = \text{const.}$) have encountered severe difficulties at these points $a_0 = 0, n\pi$.

In the following, we will therefore concentrate on the non-perturbative lattice-calculation of an effective potential for the Polyakov loops, extracting the Debye-mass from its curvature at the minimum subsequently.

3 The method of asymmetric lattices

Although huge efforts have been done to calculate an effective potential for the Polyakov loops beyond perturbative methods, no satisfying success has been reached yet except for the lattice approach. In this part, we will use the calculation of the effective action on an asymmetric lattice as described in [7].

The essential idea of this method is the fact, that on a highly asymmetric lattice with $N_t \ll N_s$ the lattice coupling β is replaced by two different spacelike and timelike couplings $\beta = \rho\beta_s = \frac{\beta_t}{\rho}$, where ρ is the asymmetry parameter $\rho = \sqrt{\frac{\beta_t}{\beta_s}}$. The quantum corrections [13] to these equations will not be considered here. Using the abovementioned equations and the fact, that on a highly asymmetric lattice $\beta_t \gg \beta_s$, a perturbative series of the action

$$S_W = \sum_{\vec{x}, i} \left(\sum_i \beta_t \frac{1}{N} \text{Tr} G_{0i}(\vec{x}, t) + \sum_{i < j} \beta_s \frac{1}{N} \text{Tr} G_{ij}(\vec{x}, t) \right). \quad (8)$$

in the spacelike coupling β_s can be performed. Only considering the order $O(\beta_s^0)$ for the moment, the spacelike link-variables $U_i(\vec{x}, t)$ of the field strengths originating at \vec{x} in direction i, j respectively

$$\begin{aligned} G_{0i}(\vec{x}, t) &= V(\vec{x}, t) U_i(\vec{x}, t+1) V^\dagger(\vec{x}+i, t) U_i^\dagger(\vec{x}, t), \\ G_{ij}(\vec{x}, t) &= U_i(\vec{x}, t) U_j(\vec{x}+i, t+1) U_i^\dagger(\vec{x}+j, t) U_j^\dagger(\vec{x}, t), \end{aligned} \quad (9)$$

can be integrated. One thus obtains [7]

$$\begin{aligned} \exp(S_{eff}^0) &= \int \prod_{x, i, t} \left[DU_{x, i, t} \left(1 + \sum_{j=\frac{1}{2}}^{\infty} d_j \frac{I_{2j+1}(\beta_t)}{I_1(\beta_t)} \chi_j(U_{x, i, t} V_{x+i, t} U_{x, t+1, i}^\dagger V_{x, t}^\dagger) \right) \right] = \\ &= \prod_{x, i} \left(1 + \sum_{j=\frac{1}{2}}^{\infty} \left[\frac{I_{2j+1}(\beta_t)}{I_1(\beta_t)} \right]^{N_t} \chi_j(P_{x+i}) \chi_j(P_x^\dagger) \right) \end{aligned} \quad (10)$$

with χ_j the characters in j th representation of the Polyakov loops

$$P_x = \begin{pmatrix} e^{i\theta_x} & 0 \\ 0 & e^{-i\theta_x} \end{pmatrix}. \quad (11)$$

This result, first obtained by [7] cannot be simplified analytically for general N_t . This is why we used $\theta_x = \theta_{x+i}$ to calculate the effective potential

$$V_{eff} = -\frac{1}{V} \ln \left(\left(\prod_x \sin^2(\theta) \right) \exp(S_{eff}^0) \right) \quad (12)$$

for general N_t .¹ The $O(\beta_s^0)$ also coincides with the result given in [14] for a strong coupling approximation. The reason for this to be the case is the fact that the $O(\beta_s^0)$ effectively amounts to neglecting the magnetic parts of the field strength. This is exactly how the result [14] has been obtained.

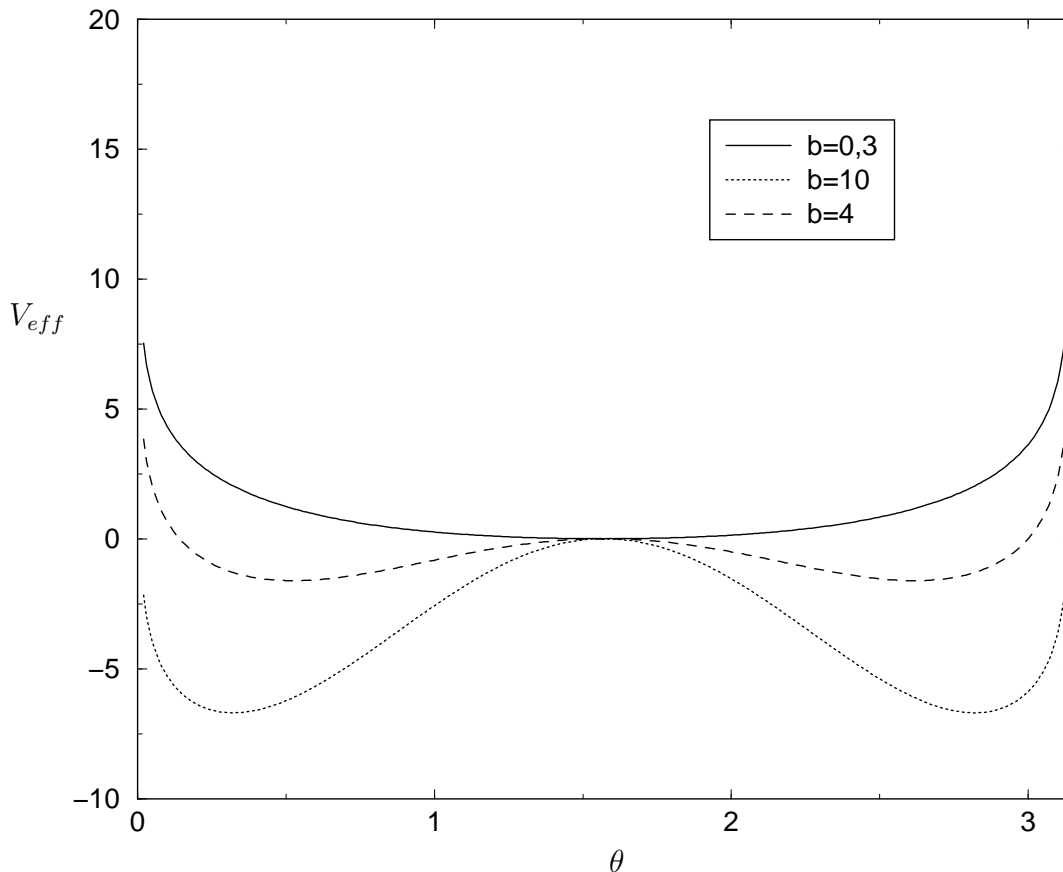


Figure 1: Potential V_{eff} in approximation $O(\beta_s^0)$ for different couplings b with a timelike extension N_t of the lattice. For increasing coupling b (increasing temperature), the potential shows a second order phase transition to a phase, where the center symmetry is spontaneously broken.

4 Numerical results

We now proceed investigating the properties of the effective potential numerically. We obtain the effective potential for the Polyakov Loops numerically by only considering spatially constant field configurations $\theta_x = \theta_{x+i}$.

¹A mean field analysis as performed in [7] for the critical coupling yields the same qualitative phase structure but is less practical for calculating the curvature of the potential at the minimum.

Then we search for the minima of the effective potential as a function of β_t for different values of N_t . After that, we calculate the curvature at the minimum by twice taking the derivative of V_{eff} with respect to the argument θ at the minimum. Next, we relate the lattice coupling to the physical temperature $\frac{T}{T_c}$ and fit the data to the functional form of the leading order (one loop) perturbative result for the Debye Mass

$$m_D = C \sqrt{\frac{2}{3}} g(T) T. \quad (13)$$

This procedure deserves a more detailed explanation.

We are eventually interested in the prefactor C . Previous lattice calculations [3], [4] suggested a $C \in [1.6, 2]$ as compared to the one loop perturbative result $C = 1$.

For being able to make reliable predictions about the size of this prefactor, one has to compare results at the same physical temperature. We follow here an approach outlined in [16].

Violations of asymptotic scaling are taken into account by considering the general Ansatz

$$a\Lambda = R(g^2)\lambda(g^2) \quad (14)$$

$$R(g^2) = \exp\left(\frac{-b_1}{2b_0^2} \ln(b_0 g^2) - \frac{1}{2b_0 g^2}\right), \quad (15)$$

$$b_0 = \frac{11N_c}{48\pi^2}, \quad b_1 = \frac{34}{3} \left(\frac{N_c}{16\pi^2}\right) \quad (16)$$

The function $\lambda(g_t^2)$ parametrizes the asymptotic scaling violations, the values for b_0 and b_1 are found perturbatively. For $\lambda(g_t^2)$, we use an exponential ansatz

$$\lambda(g_t^2) = \exp\left(\frac{1}{2b_0^2}(d_1 g_t^2 + d_2 g_t^4 + d_3 g_t^6)\right). \quad (17)$$

With $T = \frac{1}{N_t a_t}$, we obtain

$$\frac{1}{N_t R(g_{t,c}^2)} = \lambda(g_{t,c}^2) \frac{T_c}{\Lambda}. \quad (18)$$

The value of the coupling $g_{t,c}^2$ at the critical Temperature T_c of the deconfinement phase transition can be obtained numerically by inspection of the effective potentials for various N_t (see figure 1). We thus arrive at the fit parameters $d_1 = 0.0068$, $d_2 = -0.001173$, $d_3 = 6.4818 \cdot 10^{-5}$ in equation (17).

Now we can extract the temperature in units of the critical temperature for a given N_t . It is given by

$$\frac{T}{T_c} = \frac{R(g_{t,c}^2)\lambda(g_{t,c}^2)}{R(g_t^2)\lambda(g_t^2)}. \quad (19)$$

N_t	β_c	$\frac{m_D}{m_D^{p.t.}}$	$\frac{T_c}{\Lambda_{\overline{MS}}}$
1	0,39	1,98	1,013
2	1,29	1,81	0,889
3	2,09	1,80	0,983
4	2,54	1,64	1,156
5	3,18	1,64	1,330
8	5,37	1,48	1,556

Table 1: Results

This sets us in the position to relate the Temperature T to the coupling β_t . This temperature scale can now be used to relate the lattice Debye mass given by the curvature of the effective potential (12) at the minimum in terms of the asymmetric lattice coupling β_t to the physical temperature T .

We now use the functional dependence of the Debye-mass on $g(T)$ and T as known from one-loop perturbation theory to determine C in equation (13). To this end, we need the coupling as a function of the temperature $g(T)$. For this running coupling, we use the two-loop formula

$$g(T) = \left\{ \frac{11}{12\pi^2} \left(\ln \left(2\pi \frac{T}{T_c} \frac{T_c}{\Lambda} \right) \right) + \frac{17}{44\pi^2} \left(\ln \left[2 \ln \left(2\pi \frac{T}{T_c} \frac{T_c}{\Lambda} \right) \right] \right) \right\}^{-\frac{1}{2}}. \quad (20)$$

Using (14), (19) and (20) in (13), we can determine the prefactor C which gives the quotient of m_D with the perturbative value $m_D^{p.t.}$.

Carrying out this procedure for several values of N_t in the temperature range $2 < \frac{T}{T_c} < 8$, we arrive at the values for the prefactor C in the one loop perturbative Debye mass given in table 1. These results are shown in figure 2. The values for $\frac{T_c}{\Lambda_{\overline{MS}}}$, that we obtained with our fit are also indicated in the table. A comparison with the numerical result $\frac{\Lambda_{\overline{MS}}}{\Lambda_W} = 19.8231$ [15] has been performed. Thus the deviation from one gives the degree of accuracy and shows, that it agrees well with the expectation for small lattices, whereas it strongly disagrees for larger lattices. This will also be addressed in the next section.

5 Discussion and Outlook

Using a definition of the Debye-Mass which allows to extract it from the gauge invariant effective potential for the Polyakov loops, we showed by explicit solution, that the perturbative result is reproduced, when the Weiss-Potential is considered. We then used an asymmetric lattice to determine the effective potential nonperturbatively. To this end, we only considered the leading order contribution in an

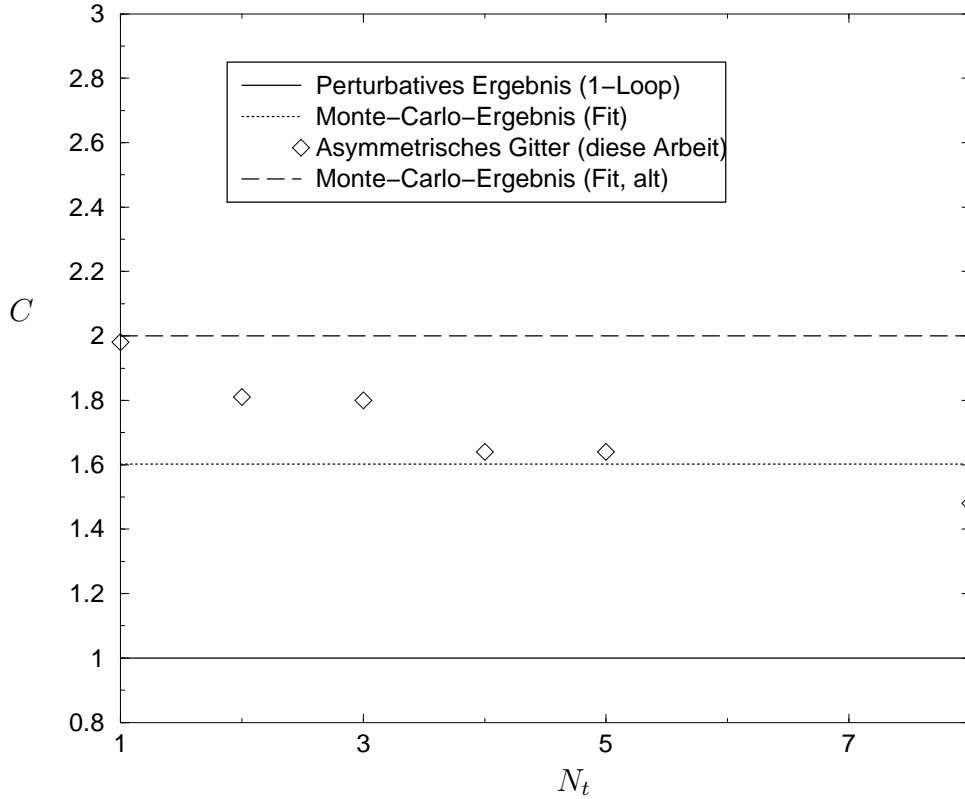


Figure 2: Comparison of the Debye masses obtained for various values of N_t . The Value of [3] and the older one of [4] as well as the perturbative value are indicated, too.

expansion in the space-like lattice coupling. This approximation is believed to be of good accuracy for highly asymmetric lattices. This can also be shown numerically by inspection of the curvatures of the corresponding effective potentials in $O(\beta_s^0)$ and $O(\beta_s^2)$ for different values of the asymmetry parameter ρ (which ultimately give the Debye mass). The result is depicted in figure 3. The influence of the next to leading order on the temperature scale is not taken into account here.

Our investigation then proceeded by numerical calculation of the Debye-Mass. The results of our calculation yield a prefactor which is significantly larger than the perturbative result. It is within the error bars due to the fits, that we performed compatible with the result suggested by full Monte-Carlo calculations [3], which is also shown in figure 2.

The major shortcoming of the method used here is the fact, that no Monte-Carlo values of $\frac{T_c}{\Lambda}$ are available for highly asymmetric lattices (only these can be hoped to give reliable predictions for the Debye-Mass). Thus there is no control as to whether the temperature scale is of good accuracy and whether the results for C are reliable. In addition to that, since we used the $O(\beta_s^0)$ of the asymmetric expansion only, we are effectively working in the strong coupling regime and neglecting magnetic

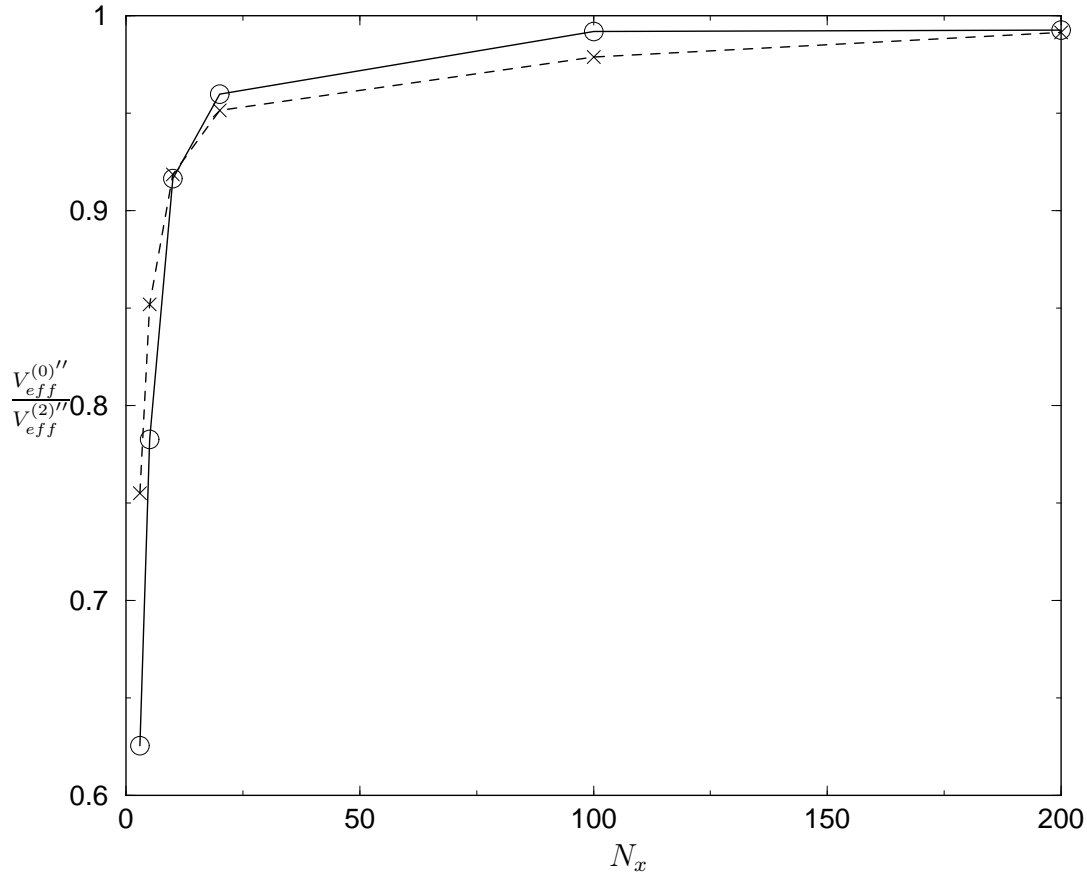


Figure 3: Difference of the curvature of the effective potentials in $O(\beta_s^0)$, $V_{eff}^{(0)''}$, and $O(\beta_s^2)$, $V_{eff}^{(2)''}$, at the minimum for $N_t = 5$ (full line) and $N_t = 3$ (dashed line), $\frac{\beta}{\beta_c} = 11, 5$. For $\rho = \frac{N_x}{N_t} = 1$, the deviation is 22% at $N_t = 5$ and 24% at $N_t = 3$, for $\rho = 20$ only 0,81% and 3,8% respectively.

terms in the action. Due to this, our results are not obtained in the scaling regime, which can also be seen from the function $\lambda(g_t^2)$ (17), which fits the asymptotic scaling violation and is not constant. This behaviour of the function parametrizing the asymptotic scaling violation is also seen in full numerical investigations [3][4]. However, this problem can easily be overcome by incorporating the contribution of the $O(\beta_s^2)$ terms. Our investigation of the errors of the truncation of the series in β_s shows, that the effect on the Debye Mass will be small on the scale of the discrepancy between perturbative continuum result and full lattice Monte-Carlo result. Furthermore, we had to do several fits, which are of varying accuracy. In this particular case, we performed the analysis for rather small values of β_t , because the expansion of the action in β_s is better for small β . In addition to that, we used perturbative results for $R(g^2)$ as well as for $g(T)$. It is unclear, whether the coupling range that we investigated is in the perturbative regime already.

On the other hand, the results show, that there is a strong influence of the wall at $PL \approx \pm 1$ in the effective Potential for the PL associated to the non-trivial Haar-measure on the size of the inverse screening length. Since the fact, that this wall does not vanish is a major property of non-perturbative treatments of the action (as has also been seen in a path-integral approach in $1 + 1$ dimensions [20] and in a new, remarkable calculation [21]), the difference in size of the Debye-Mass can be attributed to this Haar-measure, which inevitably modifies the curvature at the minimum of the effective potential.

Acknowledgements

I would like to thank the Studienstiftung des Deutschen Volkes for a PhD research grant.

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